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Can Average Calculus Students Solve Nonroutine Problems?

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Presently, there is considerable interest nationwide in improving calculus teaching due, in part, to students' inability to use it effectively in the client disciplines. Since not every application can be made routine, ability to apply calculus may be linked to students' problem-solving skills. This paper reports on a small benchmark study done in Spring 1988. We investigated whether C students from our traditional first-quarter calculus course could solve cognitively nontrivial problems, that is, problems for which they had not been taught a method of solution. Paid volunteers from various sections were tested. We describe the students, the test, and the results and raise further questions. Notably, not a single student solved an entire problem correctly and most solution attempts relied heavily on earlier, more elementary, mathematics.

INTRODUCTION

This paper is a report on a small benchmark study[1] of the title question.

What we are calling a nonroutine or novel problem is simply called a problem, as opposed to an exercise, in problem-solving studies (Schoenfeld, 1985). Most of what appears in calculus texts at the end of sections should more appropriately be called exercises. A problem in the sense used here can be regarded as having two components: a task and a solver, usually a person, but possibly a group of persons or a machine. Solving the problem consists of finding a method of solution, possibly an algorithm, and carrying it out. The solver comes equipped with information and skills, perhaps including misconceptions, for attempting the task. Although studies of problem solving do not always mention the solver, the solver is an implicit part of this view of a problem. Such studies of problem solving assume the tasks are what we will call *cognitively nontrivial*, that is, the solver does not begin knowing a method of solution. This means that novel problems cannot be solved twice by the same individual, as the second time he would already possess such a method.

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Traditional calculus courses contain few cognitively nontrivial problems. Instead, they aim to equip students to carry out as many kinds of tasks as possible. In order to do this, the tasks are divided into small parts, and algorithms, sample solutions, and examples are provided whenever possible. Indeed, many problems can be made routine in this way, and calculus texts have done so very successfully. Often students perceive cognitively nontrivial problems to be essentially unsolvable by normal people (Schoenfeld, 1985, pp. 43, 373) and consider them inappropriate (unfair) in a calculus course.

Presently, there is considerable interest in alternatives to the traditional way of teaching calculus (Douglas, 1986; Steen, 1987). Before designing such alternative courses, it would be useful to know as much as possible about the competencies of students completing the traditional course. Problem-solving ability is one competency worth considering because applications arising from outside the classroom are likely to involve at least some degree of problem solving. The inability to use calculus in applications has been mentioned as one difficulty with the present course (Steen, 1987, pp. 32, 42, 55, 95).

Although average calculus students' limited problem-solving ability probably will not surprise most calculus teachers, we decided to try to ascertain the degree of this ability. For this purpose, we prepared a test, which was nonroutine for our students, and administered it to a sample of C students from our first calculus course. What follows is a detailed description of the course, the students, the test, and the results.

THE COURSE

Tennessee Technological University is a comprehensive state university, with an engineering emphasis, enrolling about 7500 students. The average ACT composite score of the 1284 entering freshman in Fall 1987 was 20.29, above average for the state.

The course was the first quarter of our mainstream calculus, which serves all students wanting to take calculus, except those in the College of Business Administration, who are offered a separate course. It met 5 hours per week for 10 weeks in Fall Quarter 1987 and was taught from Swokowski, *Calculus with Analytic Geometry*, Alternate Edition, Chapters 1–4. Each of the eight sections consisted of about 32 students and was taught by an experienced full-time mathematics faculty member. Each professor handled his own examination and grades. Each section was taught according to the professor's normal methods; none was experimental or unusual. The engineering students, the bulk of our clientele in this course, had taken the MAA Calculus Readiness Test and had been advised accordingly. Additionally, the catalog states that an ACT math score of at least 26 is required for direct entry into the course, although this is not always enforced. Overall there were 18.5% A's, 26.6% B's, 19.3% C's, 6.2% D's, 23.6% F's, and 5.8% W's given in this course.

THE STUDENTS

During the middle of Winter Quarter, the 50 students who completed the first quarter of calculus with a C were contacted by mail and invited to participate in the study. Each student was offered a fee of $10 for taking the test and told he need not, in fact, should not study for it. Liberal prizes were offered for the top scores as an additional incentive, and to ensure that students would be motivated to do their best. In addition, they were informed that only C students were to be tested, so they would not be discouraged by lack of confidence or excessive competition. They were told nothing about the test except that it would involve first calculus.

Surprisingly, only 11 students responded to the original mailing, so the test was canceled. Early in Spring Quarter the offer was repeated. This time the test was scheduled very carefully to maximize student convenience and the fee was raised to $15, about 3 times normal earnings for an hour's part-time work in this small city. The invitation stated that we would randomly select two groups of 10 students and in each group there would be four prizes ranging from $5 to $20. The students were also informed that the results would be kept confidential and names would only be used to award prizes.

This time 20 students volunteered and 17 actually took the test. Even allowing for some scheduling and communication difficulties, this is less than half the available pool of C students. This suggests the surprising possibility that a high percentage of average calculus students have a remarkable aversion to the subject. It is hard to imagine such students voluntarily studying enough calculus outside of class to grasp the essential concepts and manipulative skills.

The 17 students who took the test had a mean ACT composite score of 24.18, and at the time they took the test, their mean quality point average was 2.539. In Winter Quarter, 15 of the 17 continued with calculus and received 13% A's, 33% B's, 13% C's, 20% D's, 7% F's, and 13% W's. The 33 C students who did not take the test had a mean ACT composite score of 25.33, and at the time the test was given, had a mean QPA of 2.558. In Winter Quarter, 24 of the 33 continued with calculus and received 4% A's, 33% B's, 29% C's, 8% D's, 17% F's, and 8% W's.

THE TEST

The test consisted of the following five problems which could be solved using concepts and skills taught in our first quarter of calculus. To ensure that the problems were nonroutine for these students, department faculty were invited to an informal seminar to consider possible problems for inclusion and to make suggestions. As far as we were able to ascertain, these problem types were not taught or assigned in any of the classes. They require students to combine what should be familiar techniques and concepts in a new way, but their solutions are no more complex than those of many sample problems covered in the course.

1. Find values of a and b so that the line $2x + 3y = a$ is tangent to the graph of $f(x) = bx^2$ at the point where $x = 3$.
2. Does $x^{21} + x^{19} - x^{-1} + 2 = 0$ have any roots between -1 and 0? Why or why not?
3. Let $f(x) = \begin{cases} ax, & x \leq 1 \\ bx^2 + x + 1, & x > 1. \end{cases}$ Find a and b so that f is differentiable at 1.
4. Find at least one solution to the equation $4x^3 - x^4 = 30$ or explain why no such solution exists.
5. Is there an a so that $\lim_{x \to 3} \frac{2x^2 - 3ax + x - a - 1}{x^2 - 2x - 3}$ exists? Explain your answer.

Before the test commenced, students were cautioned they might find some of the problems a bit unusual. They were asked to write down as many of their ideas as possible because this would be helpful to us and to their advantage. They were assured that all prizes would be awarded and that partial credit would be given. They were reminded that they were all equally matched in ability at calculus.

Each problem was printed on a separate page on which all work was to be done. The test lasted 1 hour and every student appeared to be working diligently during the whole time. Each problem was assigned 20 points and graded by one of the authors and checked by the others. Grading was reasonably liberal when compared to that of most calculus teachers or the Advanced Placement Calculus Test. The highest score was 35 out of 100.

THE RESULTS

Not one student got an entire problem correct. Most couldn't do anything. The 85 solution attempts fell naturally into two classes. In 79 attempts, students were judged as not making any reasonable progress toward a correct solution. In the remaining 6 attempts, students showed substantial progress toward a correct solution; their attempts could have been altered or completed to arrive at a correct solution and were judged at least half correct, that is, assigned 10 or more points. These six partially correct solutions were contributed by five students; Problem 1 was worked once, Problem 4 was worked twice, and Problem 5 was worked three times. The three partially correct solutions to Problem 5 called on L'Hôpital's rule. Since eight of the tested students were taking the third quarter of calculus, where L'Hôpital's rule had been covered, the problem may have been less novel than originally intended.

Of the 68 solution attempts on the first four problems, 40 made no use of calculus whatever and 11 made only perfunctory use of calculus, for example, taking a derivative and ignoring it in the remaining work. It is not surprising that a few solution attempts would rely entirely on earlier, more elementary, mathematics. However, we were astounded that 75% of these attempts made essentially no use of calculus on a test called a calculus test and in which half the problems contained the key words "tangent" and "differentiable." We omitted Problem 5 from this analysis because its attempted solutions were difficult to categorize.

The favored method of solution to Problem 1, given by seven students, was to solve the equations of the line and parabola simultaneously, thereby not using the crucial information that the line was tangent to the parabola. On Problem 2, nine students used trial and error, that is, substituted a few numbers for x to see what happens to the size of $x^{21} + x^{19} - x^{-1} + 2$ between -1 and 0 and then made a guess. On Problem 3, nine students set the two formulas, ax and $bx^2 + x + 1$, equal and guessed what a and b should be. On Problem 4, four students factored $4x^3 - x^4$ and set each factor equal to 30, and five students substituted a few values for x in $4x^3 - x^4$ and guessed. On Problem 5, eight students substituted $x = 3$ in the function, found the denominator was 0 and either didn't continue or concluded the limit could not exist because one can't divide by zero.

CONCLUSIONS AND QUESTIONS

This study indicates that many students who pass calculus cannot solve nonroutine calculus problems. Since not every application can be made routine, improving problem-solving ability might well enhance the usefulness of calculus courses. Nonroutine problems could be added to a traditional course to provide problem-solving practice. A more efficient change, however, might be to remove some explanations and examples, thereby converting existing exercises into nonroutine problems. This would avoid adding material to an already crowded syllabus (Douglas, 1986).

At the Washington Colloquium on *Calculus For A New Century*, many technical difficulties dealing with the present teaching of calculus were highlighted in the discussion groups: large lecture sections, use of inexperienced teachers and teaching assistants, poorly prepared students, lack of proper placement tests. Our students suffered none of these handicaps, yet came away from the course with little knowledge that they could use in solving novel problems. Thus, remedying such handicaps, though laudable, is unlikely by itself to improve students' problem-solving abilities or satisfy client disciplines.

This study suggests several natural questions. How would the A and B students have performed? We had anticipated that they would be able to apply their knowledge flexibly. It turns out, however, that 2 of our 17 volunteers earned A, and 9 earned B, in the second quarter of calculus. This and the extreme nature of our results suggests the better students should be studied.

Which of our results were due to deficiencies in students' problem-solving techniques (heuristics and control; Schoenfeld, 1985) as opposed to their inadequate knowledge base? It is possible that C students really know less than we thought. In a repeat study one could include a section of routine exercises to check for the knowledge needed in the novel problems.

Do average calculus students really has as strong an aversion to calculus as the response rate to our study suggested? This could be tested by comparing students' willingness to take a calculus test as opposed to their willingness to volunteer for some more neutral task.

Finally, how many calculus students view mathematics as static and consisting largely of standard procedures for working routine problems? This fundamental misconception of the nature of mathematics may be widely held by students, and for that matter, by some teachers. Students holding this view are likely to resist any emphasis on cognitively nontrivial problems, regarding them as distractions and irrelevant to the main business of a calculus course.

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