Eurocentrism in the History of Mathematics: the Case of the Kerala School

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To a typical historian of mathematics today, if there is one certainty, it is that Isaac Newton (1642–1727) and Gottfried Leibniz (1646–1716) were the first to 'invent' a generalised system of infinitesimal calculus, an essential prelude to modern mathematics. However, at least two hundred years earlier, the astronomer-mathematicians of Kerala, notably Madhava of Sangamagrama and his disciples, had discovered elements of that calculus, the forerunners of modern techniques used in mathematical analysis. Given the existence of a corridor of communication between Kerala and Europe, especially from the sixteenth century onwards, and the crucial importance of calculus in the growth of modern mathematics, one would have expected that the possibility of the transmission of the Kerala mathematics westwards would be high on the agenda for historical investigation. That such an investigation has not yet been carried out may reflect, in our view, the strength and the pervasive nature of Eurocentrism in the history of science.

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Introduction

From early times, Indian mathematics was often bundled together with astronomy and astrology, a fact recognised and commented on by scientists such as Al-Biruni as early as the eleventh century AD.[1] So it is not difficult to understand why the mathematics of the Kerala School[2] was initially developed by astronomers to service astrological prediction and facilitate the construction of a precise calendar. This mathematics included numerical integration methods and infinite series derivations for certain trigonometric functions and was transcribed in palm leaf manuscripts by Kerala astronomers such as Nilakantha Somayaji, Jyesthadeva, Sankara Variyar and others between the fourteenth and sixteenth centuries.[3] These astronomers had elaborated and extended the earlier work of the master astronomer and founder of the Kerala School, Madhava of Sangamagrama (c.1340–1425).

It is important to bear in mind that the conceptual and epistemological bases of Madhava's mathematics had little affinity with those of early Greek mathematics. Instead, they were founded on the principles elaborated in 499 AD by the renowned Indian astronomer Aryabhata in his *Aryabhatiya*. While Aryabhata and his predecessors may have been influenced by the *content* of classical Greek mathematics they did not subscribe to the idea of Platonic reality[4] and deductive proof, the essential characteristics of the Greek contribution. Instead, the Indians subscribed to a paradigm in which the truth of mathematical results could be established by both empirical and abstract reasoning (or *pramana*). No conflict was perceived between *pramana*, on the one hand, and empirical demonstration or numerical calculation, on the other. Deduction was an integral part of *pramana*, but it was not imagined, as in the case of Greek mathematics, that the exclusion of the empirical somehow conferred a superior and infallible status on deduction. In addition, the use of irrational numbers, unlike early Greek mathematics, was accepted in Indian mathematics by the use of floating point number approximations.[5]

It is noteworthy that the reluctance of Indian astronomers to accept Greek knowledge unconditionally had its parallels in Europe. The Portuguese and Spanish explorers of the fifteenth and sixteenth centuries cast a critical eye over the immutable laws of antiquity and their insistence on discovery through thought experiment.[6] The Iberian explorers were justified in their scepticism when, in their voyages of discovery, they found certain aspects of the astronomy of Ptolemy and Hipparchus to be incorrect. This is not to say that the Portuguese explorers rejected ancient knowledge; instead they, like the Indians, modified it to suit their own particular ideas. This is most aptly demonstrated by Portugal's greatest medieval scientist Pedro Nunes, who considered Ptolemy's geography to be intrinsically flawed, but nevertheless modified it to develop the concept of the *loxodrome* as an aid for navigation.[7]

In a similar vein, the Aryabhatan School of Indian mathematics was justified in its epistemological approach when, in approximately the same period as the Iberian explorers, Madhava and his students used procedures involving the passage to infinity and *pramana* – things alien to the Greek mathematical epistemology – to discover infinite series sums for various functions. The Madhava School was comfortable in dealing with the concept of infinity and, moreover, applied this concept in a sophisticated way to be replicated in the period following the development of general methods of the calculus by Leibniz and Newton. As Rajagopal and Marar point out:

> There are two points which emerge from a consideration of the [mathematics of the Kerala School] . . . In the first place, it employs relations which would appear not to have been noticed in Europe before modern forerunners and followers of the calculus started investigations . . . Our second point is not unconnected with the first. The Hindu mathematicians achieved, without the aid of calculus, results which, for us, are treated best by means of the calculus . . . This is not to gainsay the fact that (i) the Hindus' proof of [infinite] series shows their awareness of the principle of integration as we ordinarily use it nowadays (ii) their intuitive perception of small quantities . . . is as good as a practical knowledge of differentiation.[8]

This line of argument suggests that the Renaissance mathematicians of Europe, who were schooled in the mathematics of Euclid and who came before 'modern forerunners and followers of the calculus started investigations', may have had some difficulty with the concept of infinity. The controversies over Cavalieri's *indivisibles* and Newton's *fluxions* suggest this to be the case.[9] On a prototypical level, we can give an example of this awkwardness in the work of the prominent seventeenth-century English mathematician John Wallis. When Wallis attempted to generalise the result '. . . $1/5 < 1/4 < 1/3 < 1/2 < 1/1$', induction led him to conclude that '$1/0 < 1/-1 < 1/-2 < 1/-3 < 1/-4 < \ldots$'. This erroneous conclusion showed a faulty understanding of the transition from positive quantities to negative ones via infinity. Wallis did later formulate a prototypical concept of infinitesimally small quantities, but the awkwardness remained.[10]

This awkwardness in dealing with infinitesimals (infinitely or extremely small quantities) and, by implication, with infinity exhibited by Renaissance mathematicians was most likely due to their schooling in Greek mathematics, with its reticence over dealing with such concepts. It is a matter of conjecture whether they would have developed the calculus earlier had they embraced other mathematical epistemologies besides the Greek.

Foreign perspectives on Indian and Kerala mathematics in the second millennium

In the early part of the second millennium, evaluations of Indian mathematics or, to be precise, astronomy were generally from Arab commentators. They tended to indicate that Indian science and mathematics were independently derived. Some, like Said Al-Andalusi, claimed them to be of a higher order:

> [The Indians] have acquired immense information and reached the zenith in their knowledge of the movements of the stars [astronomy] and the secrets of the skies [astrology] as well as other mathematical studies. After all that, they have surpassed all the other peoples in their knowledge of medical science and the strengths of various drugs, the characteristics of compounds, and the peculiarities of substances.[11]

Others, like Al-Biruni, were more critical. For him, Indian mathematics and astronomy were much like the vast mathematical literature of the twenty-first century – uneven with a few good quality research papers and a majority of pedestrian or error-strewn publications: 'I can only compare their mathematical and astronomical literature, as far as I know it, to a mixture of pear shells and sour dates, or of pearls and dung, or of costly crystals and common pebbles.'[12]

Nevertheless, a common element in these early evaluations is the uniqueness of the development of Indian mathematics. However, by the nineteenth century and contemporaneous with the establishment of European colonies in the East, the views of European scholars about the supposed superiority of European knowledge had developed racist overtones. Sedillot asserted that not only was Indian science indebted to Europe, but also that Indian numbers were an 'abbreviated form' of Roman numbers, that Sanskrit was 'muddled' Greek and that India had no chronology.[13] Although Sedillot's assertions were based on imperfect knowledge and understanding of the nature and scope of Indian mathematics, this did not deter him from concluding:

> On one side, there is a perfect language, the language of Homer, approved by many centuries, by all branches of human cultural knowledge, by arts brought to high levels of perfection. On the other side, there is [in India] Tamil with innumerable dialects and that Brahmanic filth which survived to our day in the environment of the most crude superstitions.[14]

In a similar vein, Bentley also cast doubt on the chronology of India by locating Aryabhata and other Indian mathematicians several centuries later than was actually the case. He was of the opinion that Brahmins had actively fabricated evidence to locate Indian mathematicians earlier than they existed:

> We come now to notice another forgery, the *Brahma Siddhanta Sphuta*, the author of which I know. The object of this forgery was to throw Varaha Mihira, who lived about the time of Akber, back into antiquity . . . Thus we see how Brahma Gupta, a person who lived long before Aryabhata and Varaha Mihira, is made to quote them, for the purpose of throwing them back into antiquity . . . It proves most certainly that the *Brahma Siddhanta* cited, or at least a part of it, is a complete forgery, probably framed, among many other books, during the last century by a junta of Brahmins, for the purpose of carrying on a regular systematic imposition.[15]

For the record, the actual dates are Aryabhata b. 476, Varamihira existed *c*. 505, Brahmagupta *c*. 598, and Akbar *c*. 1550.[16] So it is safe to suggest that Bentley's hypothesis was an indication of either ignorance or a fabrication based on a Eurocentric view[17] of the history of science. Nevertheless, Bentley's altered chronology had the effect not only of lessening the achievements of Indian mathematics, but also of making redundant any conjecture of transmission to Europe.

Inadequate understanding of Indian mathematics was not confined to run of the mill scholars of a period long past. More recently, D. E. Smith, an eminent historian of mathematics, claimed that, without the introduction of western civilisation in the eighteenth and nineteenth centuries, India would have stagnated mathematically. He went on to say that: 'Not since Bhaskara [Bhaskara II, b. 1114] has she produced a single native genius in this field.'[18]

It is clear that Smith was either unaware of or ignored the works of contemporary scholars such as Whish and Warren, who were the first Europeans to acknowledge the achievements of the Kerala School.[19] This fashion of ignoring these advances has persisted until even very recent times, with no mention of the work of the Kerala School in Edwards's text on the history of the calculus,[20] nor in articles on the history of infinite series by historians of mathematics such as Abeles and Fiegenbaum.[21] A possible reason for such puzzling standards in scholarship may have been the rising Eurocentrism that accompanied European colonisation.[22] With this phenomenon, the assumption of white superiority became dominant over a wide range of activities, including the writing of the history of mathematics. The rise of nationalism in nineteenth-century Europe and the consequent search for the roots of European civilisation led to an obsession with Greece and the myth of Greek culture as the cradle of all knowledge and values, with Europe as heir to Greek learning and values.[23] As Bernal has shown, in the 'Greek miracle' the Afro-Asiatic roots of Greece were virtually buried.[24] What emerged as an account of the historical development of mathematical knowledge was an unreconstructed Eurocentric trajectory that ignored or devalued the contribution of the rest of the world. Rare exceptions to this skewed version of history were provided by Ebenezer Burgess and George Peacock. They, respectively, wrote:

> Prof. Whitney seems to hold the opinion, that the Hindus derived their astronomy and astrology almost bodily from the Greeks . . . I think he does not give the Hindus the credit due to them, and awards to the Greeks more credit than they are justly entitled to.[25]

> [It] is unnecessary to quote more examples of the names even of distinguished men who have written in favour of a hypothesis [of the Greek origin of numbers and of their transmission to India] so entirely unsupported by facts.[26]

However, by the latter half of the twentieth century, European scholars, perhaps released from the powerful influences induced by colonisation, had started to analyse the mathematics of the Kerala School using largely secondary sources such as Rajagopal and his associates and Saraswati Amma.[27] The achievements of the Kerala School and their chronological priority over similar developments in Europe were now being aired in several western publications.[28] However, these evaluations are accompanied by a strong defence of the European claim for the invention of the generalised calculus.

For example, Baron states that:

> The fact that the Leibniz-Newton controversy hinged as much on priority in the development of certain infinite series as on the generalisation of the operational processes of integration and differentiation and their expression in terms of a specialised notation does not justify the belief that the [Keralese] development and use for numerical integration establishes a claim to the invention of the infinitesimal calculus.[29]

Calinger writes:

> Kerala mathematicians lacked a facile notation, a concept of function in trigonometry . . . Did they nonetheless recognise the importance of inverse trigonometric half chords beyond computing astronomical tables and detect connections that Newton and Leibniz saw in creating two early versions of calculus? Apparently not.[30]

These comparisons appear to be defending the roles of Leibniz and Newton as inventors of the generalised infinitesimal calculus. While we understand the strength of nationalist pride in the evaluation of the achievements of scientists, we do find difficulty in the qualitative comparison between two developments founded on different epistemological bases. It is worthwhile reiterating the point made earlier that the initial development of the calculus in seventeenth-century Europe followed the paradigm of Euclidean geometry in which generalisation was important and in which the infinite was a difficult issue. On the other hand, from the fifteenth century onwards, the Kerala mathematicians employed computational mathematics with floating point numbers to understand the notion of the infinitesimal and derive infinite series for certain targeted functions. In our view, it is clear that qualitatively different intellectual tools used in different eras to investigate similar problems are likely to produce qualitatively different outcomes. Thus, the sensible way to understand Kerala mathematics is to understand it within the epistemology in which it was developed. To do otherwise is akin to trying to gain a full appreciation of the literature of Shakespeare by translating it literally into Urdu – the semantic and cultural connotations would undoubtedly be lost.

Paradigms for establishing transmission and their Eurocentric applications

The basis for establishing the transmission of science is generally taken to be *direct* evidence of translations of the relevant manuscripts. With respect to the issue under discussion, the transmission of early Indian mathematics and astronomy to Europe has been established by direct evidence. The transmission of Indian computational techniques was in place by at least the early seventh century, for, by 662 AD, it had reached the Euphrates region.[31] Texts that contributed to the transmission of Indian computational techniques to Europe are discussed at length by Benedict.[32] Indian astronomy was transmitted westwards to Baghdad, by a translation into Arabic of the astronomical treatises, the *Siddhantas*, around 760 AD, and into Spain by translation into Latin of the same work in 1126.[33] This transmission was not restricted westwards for there is documentary evidence of Indian mathematical manuscripts being found and translated in China, Thailand, Indonesia and other Southeast Asian regions from the seventh century onwards.[34] Despite the continuous flow of information from India during this period, the literature is strangely silent regarding transmission during the medieval period. It is as if a tap had been turned off permanently.

In the absence of direct evidence, it is often considered to be sufficient to fulfil a set of conditions to establish that transmission occurred. According to Neugebauer, a set of sufficient conditions are: i) the identification of methodological similarities; ii) the existence of communication routes; and iii) a suitable chronology for the transmission.[35] Using this paradigm, Neugebauer established a claim about the Greek origins of the astronomy contained in the Indian astronomical texts, the *Siddhantas*. According to van der Waerden, chronological priority is considered generally sufficient in his 'hypothesis of a common origin' and, using this paradigm, he claimed that Aryabhata's trigonometry was borrowed from the Greeks.[36] In an earlier work, van der Waerden made a similar claim about the twelfth-century work of the Indian mathematician Bhaskara II, Diophantine equations[37] wherein he argues that 'the original common source of the Hindu authors was a Greek treatise in which the whole method was explained'.[38] It is interesting to note that the identity of this Greek work is not revealed and that he concludes by stating that 'in the history of science independent inventions are exceptions: the general rule is dependence'.[39]

Neither Neugebauer nor van der Waerden can claim to be the originators of the transmission criteria discussed above. These have, in some form or other, been around since the Renaissance and constitute, it may be suggested, part of a common understanding amongst scholars in Europe. For example, Sedillot used the existence of communication routes after European colonisation to argue for the transmission of mathematical and astronomical ideas from Greece[40] and Portugal[41] to India. A later historian, O'Leary, used a mixture of the Neugebauer and van der Waerden paradigms to posit the Greek origin of Indian astronomy and mathematics. He was sufficiently convinced of this to be able to state that: 'It is not necessary to prove that translations of the Greek scientists were actually made in Hindi or Persian, it is sufficiently clear that their teaching was known and used.'[42]

These examples suggest that a case for claiming the transmission of knowledge from one region to another does not necessarily rest on documentary evidence. Given the possibility that documents from ancient and medieval times may have perished or been lost due to a variety of reasons, it would seem that this methodology is sensible; that is, in the absence of direct evidence, priority, communication routes and methodological similarity form a sufficient set of conditions to establish a socially acceptable case for transmission.

However, in our view, methodological similarities assume that the epistemological foundations in the donor and recipient cultures are similar. In the case under discussion, as we have elaborated above, they are not. Thus, one may imagine that there were methodological difficulties faced in Europe in adopting ideas and concepts from Indian mathematics. The difficulties faced in the adoption of the Indo-Arabic number system and Indian arithmetic techniques in Europe are a case in point.[43] With respect to the mathematics of the

Kerala School, we could reiterate the relative awkwardness of Renaissance mathematicians in treating infinitesimally small quantities as another case in point. Thus, we would argue for identifying methodological *similarities* as well as methodological *discordances* when attempting to identify possible transmission of a particular idea. We would also argue that the existence of a necessary set of conditions for transmission is not sufficient for the phenomena to actually occur. For example, the availability of the navigation device, the *Kamal*,[44] to Iberian sailors in the medieval period was not sufficient for it to be transmitted to other European seafaring nations. In our estimation, a sufficient set of conditions may be derived from those used in a court of law; namely, the existence of motivation, opportunity and means. Thus our modified criteria for transmission in the absence of direct evidence are: *i) the existence of a corridor of communication; ii) a suitable chronology of transmission establishing priority; iii) the existence of adequate evidence of methodological similarities and discordances; and iv) motivation, opportunity and means present to facilitate this transmission.*

In any event, the application of transmission paradigms, in whatever form, has been to identify the indebtedness of East to West. To address the issue of transmission in the opposite direction we will utilise the modified transmission paradigm we have stated above. To justify why we may apply this transmission paradigm from India to Europe during the medieval period we claim that the criteria listed above have been satisfied. First, a sea communication route between Kerala and the Arabian Gulf (via the port of Basrah) had been in existence for centuries.[45] The arrival of the Portuguese Vasco da Gama to the Malabar coast in 1499 heralded a direct route between Kerala and Europe via Lisbon. Thus, after 1499, Kerala was linked directly to Europe. Second, the priority of Kerala developments in the calculus over those of Newton and Leibniz is now beyond doubt.[46] Third, we list some of the methodological similarities and discordances:

- The *Yuktibhasa* by Jyesthadeva is a key work of the Kerala School. It gives a formula involving a passage to infinity that permits the calculation of areas under parabolas – a key component of the calculus. It is well known that this same formula was used by Renaissance mathematicians such as Pierre Fermat, Blaise Pascal and John Wallis.[47] In the case of Wallis, we see a revealing discordance in the naive way he tries to justify the formula.[48]
- Nilakantha's planetary model of the fifteenth century in the *Tantrasangraha* is very similar to the model adopted by the Renaissance astronomer Tycho Brahe.[49]
- Several of Wallis's results on continued fractions are identical to those of Bhaskara II, about 500 years earlier.[50]

The *motivation* for import of knowledge from India to Europe arose from needs in medicine (for evidence of this we point to Garcia da Orta's popular *Colloquios dos simples e drogas he cousas mediçinas da India* published in Goa in 1563; see also Iannaccone (1998) for the role of the Jesuit Schreck in this area), in practical mathematics and in navigation. As Katz points out: 'as European explorers set out on voyages around the globe, they needed improved navigational techniques that could come only through correct astronomical tables.'[51] Taken together with the fact that the voyages of exploration were of supreme economic importance, it is not difficult to see why navigation became a matter of the greatest strategic importance. Indeed, lucrative prizes were offered for anyone who could provide accurate techniques of navigation, and several Renaissance mathematicians, including Newton, Galileo and Wallis, competed (unsuccessfully). In this way, there was a motivation for Europeans to seek out knowledge in the colonies about navigation techniques.

To identify the *opportunity and means* we point to the role of the Jesuits. The arrival of Francis Xavier in Goa in 1540 heralded the continuous presence of the Jesuits in the Malabar until 1670. While the early Jesuits were interested in learning the local languages and conversion work, the latter Jesuits arriving after 1578 were of a different mould. They were all mathematicians trained by Christoph Clavius or Christoph Grienberger at the Collegio Romano.[52] These Jesuits, most notably Matteo Ricci, Johann Schreck and Antonio Rubino, went to the Malabar region, including the city of Cochin, the epicentre of developments in the infinitesimal calculus.[53] Rubino and Ricci have been recorded in correspondence as answering requests for astronomical information from Kerala sources.[54]

Despite the apparent sufficiency of conditions to warrant a study of the transmission of Kerala mathematics westwards, there has been little if any academic enthusiasm to follow it through. It is true that recently some scholars have quietly raised conjecture regarding such transmission. For example, Calinger[55] tentatively raises the conjecture of transmission of Kerala mathematics to Europe and Brezinski asks if the seventeenth-century English mathematician John Wallis may have derived his results on continued fractions from the earlier work of the Indian mathematician Bhaskara:

> [I]t is highly probable that Muslim mathematicians were in close contact with their Indian colleagues during this period, and that they brought back their writings to western countries. Since Arabic mathematicians were translated much earlier into European languages, it is possible that the Indian mathematical writings were known in Europe before their translation.[56]

However, to our knowledge, prior to 2002 there has not been a single research project to investigate this hypothesis. It may be that there is a reluctance to state such a controversial hypothesis unambiguously and investigate it rigorously. But this may only be controversial in that one risks undermining the Eurocentric perception of the hegemony of Greek and European mathematics. Once this perception is removed, then the research question could be located high on the agenda in the history of science.

Conclusions

We have seen that, with some rare exceptions, the dominant view of the mathematics of India represented a drastic departure from that of both early English commentators (such as Reuben Burrow)[57] as well as even earlier Arab commentators (such as Al-Andalusi). This departure was contemporaneous with the establishment of the European colonies in the East and it is assumed that such views were consistent with the imperialist policies of the governments of the day. Additionally, there were attempts in the nineteenth century to marginalise the achievements of Indian mathematics by casting doubts on the chronology used in India (for example, by Sedillot), thereby questioning the temporal priority of the discoveries of Indian mathematicians (for example, by Bentley). This encapsulates the Eurocentric view of Indian mathematics in nineteenth-century Europe. In the twentieth century, this Eurocentric view persisted, initially by ignoring early reports by Whish (1835) and Warren (1825) of the Kerala School of Madhava and presenting a partial history of calculus (for example, Smith (1923) and Edwards (1979)). Several subsequent works acknowledge the priority of the calculus developments of the Kerala School, but sideline them by comparing their achievements to European works founded on a different epistemology some 200 years later (for example, Baron (1983), Katz (1995) and Calinger (1999)). This, taken together with the absence of a viable discussion in the academic community about the need for a research project to investigate the compelling hypothesis of the transmission of Kerala mathematics to Europe, does suggest that the Eurocentric view still pervades the history of mathematics.

From a pragmatic point of view, we have to accept the fact that almost all studies establishing the indebtedness of East to West have been conducted by westerners. This is most likely for subjective reasons, including a desire to establish the legacy of the West and improve the likelihood of attracting research funding for such projects from organisations such as the European Union, which has its own agenda of promoting a positive image of Europe. Thus, by force of these facts, the research on transmission in the opposite direction will have to find impetus from scholars from the East or with origins in the East. However, impetus must have checks and balances to maintain objectivity and it is within such a structure that the authors have embarked on a three-year research project to rigorously and objectively examine the hypothesis of the transmission of Indian mathematics from Bhaskara II onwards to Europe.

In conclusion, we maintain that the only sensible way to understand Kerala mathematics is to understand it within the epistemology in which it was developed – to do otherwise is to lose semantic, pedagogical and cultural meaning. This perception has additional implications in that it will help scholars to understand fully the tensions of earlier attested transmissions from India to Europe. And, given the growing tension today between pure mathematicians who insist upon following the proof tradition and applied mathematicians who see the enormous possibilities opened up by the mathematics of numerical computation, such a perception might offer insights into these foundational tensions in mathematics. A similar observation may be made regarding the role of proof in Indian mathematics and in modern pure mathematics, on the one hand, and the role of proof for the student and engineer, on the other.[58] However, this is yet another subject that requires detailed treatment.

References

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1 Al-Biruni, 1030, *India*, trans. Qeyamuddin Ahmad (New Delhi, National Book Trust, 1999), p. 70.

2 Madhava began a school that had the following teacher-student lineage:
Madhava (fl. 1380–1420) → *Parameswara* (fl. 1380–1460) → Damodara (fl. 1450) → *Nilakantha* (b. 1444) → Chitrabhanu (fl. 1530) → Narayana (fl. 1529) and Sankara Variyar (fl. 1556).
Also:
Damodara → *Jyesthadeva* (fl. 1500–1575) → *Achuta Pisharoti* (fl. *c*.1550, d. 1621).
The names italicised are generally recognised as the major figures of the Kerala School.

3 The mathematical and astronomical works of the Madhava (or Kerala) School were written either in Sanskrit (for example, the *Tantrasangraha* of Nilakantha) and/or Malayalam (the local language of Kerala, as, for example, the *Yuktibhasa* of Jyesthadeva). Many of these texts have not yet been studied, but several scholars with the required linguistic skills have studied parts of the key texts such as the *Tantrasangraha* and the *Yuktibhasa* and written articles for the benefit of the English-speaking academic community (for example: T. A. Saraswati Amma, *Geometry in Ancient and Medieval India* (Varanasi, Motilal Banarsidass, 1963); C. T. Rajagopal and M. S. Rangachari, 'On an untapped source of medieval Keralese mathematics', *Archive for the History of Exact Sciences* (Vol. 18, 1978), pp. 89–102; K. V. Sarma,

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4 From Plato's *Republic* to Proclus' Neoplatonism, mathematical entities always played the role of intermediaries between the immaterial realities of the highest realm of being and the confusedly complex objects of the sense world.

5 Greek difficulties with 'irrational numbers' (numbers such as the square root of two or the ratio of circumference of a circle to its diameter (π) whose values cannot be exactly determined) arose from the attempt to establish a close correspondence between geometric and arithmetic quantities, the result being a heavy emphasis on a geometric interpretation of the irrationality of numbers. Because of this geometric bias, the Greeks were not at ease with irrational numbers and consequently operations with numbers were reduced to a narrow geometric realm, robbing them of considerable potency in arithmetic. On the other hand, with the stress in the Indian tradition on operations with numbers rather than the numbers themselves, Indian mathematics steered clear of any problem with incommensurability. See G. G. Joseph, 'What is a square root? A study of geometrical representation in different mathematical traditions', in C. Pritchard (ed.), *The Changing Shape of Geometry* (Cambridge University Press, Cambridge, 2003).

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17 By a Eurocentric history of science, we mean any account of modern science that appeals solely to causes and ideas within Europe and simultaneously marginalises the growth of modern scientific ways of thinking outside of Europe.

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